2022 ICMTA Joint Meeting on On-Off-line Hybrid

International Conference on Matrix Theory with Applications to Combinatorics, Optimization and Data Science

December 1 - 5, 2022
Seogwipo KAL Hotel, Jeju, Korea

Hosted by

NRF SRC AORC Applied Algebra & Optimization Research Center

NRF National Research Foundation of Korea

SUNGKYUNKWAN UNIVERSITY 1398

SHANGHAI UNIVERSITY IRCTMT

ZHEJIANG NORMAL UNIVERSITY 1956

Georgia Tech

Information

Including Q&A time, the plenary talk is scheduled for 50 minutes and the general invited talk is scheduled for 30 minutes.

One can join this conference via zoom. The links are as follows:

- Diamond room:

  https://us02web.zoom.us/j/8623996286

- Rose room:

  https://us02web.zoom.us/j/3359521323

Opening Remarks:

- Gi-Sang Cheon (AORC)
- Qing-Wen Wang (IRCTMT)
- Xuding Zhu (CDM)

<Note>

1. All times in the program are in **Korean time (GMT+9:00)**.
2. Online speakers are requested to **link at least 30 minutes in advance**.

Timetable

2022 ICMTA Program

CMT: Combinatorics and Matrix Theory, **MTA**: Matrix theory with Applications, **ODS**: Optimization and Data Science

	12/01 Thursday	12/02 Friday		12/03 Saturday		12/04 Sunday	12/05 Monday
	Room	Diamond (Hybrid-Zoom 1)	Rose (Online-Zoom 2)	Diamond (Hybrid-Zoom 1)	Rose (Online-Zoom 2)	Diamond (Hybrid-Zoom 1)	
	Session	CMT	MTA	ODS	CMT	MTA / ODS	
08:30-08:55	.	**Opening Remarks**					
09:00-09:50		Plenary Talk Richard Brualdi		Plenary Talk Fred Roberts (online)		Plenary Talk Steve Kirkland (online)	Discussion & Check-out
09:50-10:10		Break					
10:10-10:40		Bryan Shader (online)	Ren-Cang Li (online)	Woocheol Choi		Yang Zhang	
10:50-11:20		Jang Soo Kim	Tin-Yau Tam (online)	Yeonjong Shin		Dragana Cvetkovic Ilic	
11:30-12:00		Xuding Zhu (online)	Guanjing Song (online)	Plenary Talk Sung Ha Kang (online)		Plenary Talk Grey Ballard	Farewell Meeting
12:00-12:20		**12:10 Group Photo**					
12:20-14:00		**Lunch**					
14:00-14:50	Check-in & Registration	Plenary Talk Sang-il Oum		Plenary Talk Jein Shan Chen		Free Discussion	
14:50-15:10		Break					
15:10-15:40		Ian Wanless (online)	Qing-Wen Wang (online)	Yoonmo Jung	Daqing Yang (online)		
15:50-16:20		Sang June Lee	Luyining Gan (online)	Donghwan Kim	Zhao Zhang (online)		
16:30-17:00		Samuele Giraudo (online)	Zhuo-Heng He (online)	Abdullah Alazemi	Youngjoon Hong (online)		
17:10-17:40		Geir Dahl (online)		Ernest K. Ryu (online)			
18:00-20:00	Welcome Reception					Banquet	

The timetable for each division is as follows.

: Online talk (yellow marker)

- Plenary talks

December 2, Friday			
9:00–9:50		Chair: Suk-Geun Hwang **Richard A. Brualdi** University of Wisconsin–Madison, USA	Combinatorial Matrices and Their Continuous Analogues
14:00–14:50		Chair: Soonhak Kwon **Sang-il Oum** Discrete Mathematics Group, Institute for Basic Science, Korea	Γ-graphic delta-matroids and their applications
December 3, Saturday			
9:00–9:50	Online	Chair: Suh-Ryung Kim **Fred Roberts** Rutgers University, USA	Graph-theoretical Models of the Spread and Control of Disease and of Fighting Fires
11:30–12:20	Online	Chair: Grey Ballard **Sung Ha Kang** Georgia Institute of Technology, USA	Identifying Differential Equations with Numerical Methods: Time Evolution, Subspace Pursuit and Weak Form
14:00–14:50		Chair: Yoonmo Jung **Jein-Shan Chen** National Taiwan Normal University, Taiwan	Smoothing strategies in optimization and data science
December 4, Sunday			
9:00–9:50	Online	Chair: Richard Brualdi **Steve Kirkland** University of Manitoba, Canada	Kemeny's constant for an undirected graph: how much can adding one edge change things?
11:30–12:20		Chair: Sung Ha Kang **Grey Ballard** Wake Forest University, USA	Tensor Decompositions

- Combinatorics and Matrix Theory

December 2, Friday		
10:10–10:40	Chair: Sang-il Oum **Bryan Shader** University of Wyoming, USA	Characterization of equality in a perrank bound
10:50–11:20	Chair: Sang-il Oum **Jang Soo Kim** AORC, Sungkyunkwan University, Korea	Affine Gordon–Bender–Knuth identities for cylindric Schur functions
11:30–12:00	Chair: Sang-il Oum **Xuding Zhu** Zhejiang Normal University, China	List 4-colouring of planar graphs
12:20–14:00	**Lunch**	
15:10–15:40	Chair: Gi-Sang Cheon **Ian Wanless** Monash University, Australia	Mutually orthogonal binary frequency squares
15:50–16:20	Chair: Gi-Sang Cheon **Sang June Lee** Kyung Hee University, Korea	On strong Sidon sets of integers
16:30–17:00	Chair: Gi-Sang Cheon **Samuele Giraudo** Université Gustave Eiffel, France	Clones of pigmented words and realizations of varieties of monoids
17:10–17:40	Chair: Gi-Sang Cheon **Geir Dahl** University of Oslo, Norway	The Permutation and Alternating Sign Matrix Cones
December 3, Saturday		
15:10–15:40	Chair: Jang Soo Kim **Daqing Yang** Zhejiang Normal University, China	An approximate version of the Strong Nine Dragon Tree Conjecture
15:50–16:20	Chair: Jang Soo Kim **Zhao Zhang** Zhejiang Normal University, China	Parallel Approximation Algorithm for the Minimum Submodular Cover Problem

- Matrix theory with Applications

December 2, Friday		
10:10–10:40	Chair: Yang Zhang **Ren-Cang Li** University of Texas at Arlington, USA	A Roadmap for Solving Optimization on Stiefel Manifolds from Machine Learning
10:50–11:20	Chair: Yang Zhang **Tin-Yau Tam** University of Nevada, Reno, USA	Geometry of geometric mean and spectral mean
11:30–12:00	Chair: Yang Zhang **Guanjing Song** Weifang University, China	Low Multi-Rank Third-order Tensor Completion by Riemannian Conjugate Gradient Descent Method
12:20–14:00	**Lunch**	
15:10–15:40	Chair: Dragana Cvetković Ilić **Qing-Wen Wang** Shanghai University, China	Solvability and the general solution to a Sylvester-type quaternion matrix equation with applications
15:50–16:20	Chair: Dragana Cvetković Ilić **Luyining Gan** University of Nevada, Reno, USA	Matrix Inequalities on Metric and Spectral Geometric Means
16:30–17:00	Chair: Dragana Cvetković Ilić **Zhuo-Heng He** Shanghai University, China	The complete equivalence canonical form of four matrices over the real quaternion algebra
December 4, Sunday		
10:10–10:40	Chair: Abdullah Alazemi **Yang Zhang** University of Manitoba, Canada	On the rank problems of the third-order quaternion tensors

- Optimization and Data Science

December 3, Saturday		
10:10–10:40	Chair: Sangwoon Yun **Woocheol Choi** AORC, Sungkyunkwan University, Korea	Remarks on the optimal control problems
10:50–11:20	Chair: Sangwoon Yun **Yeonjong Shin** KAIST, Korea	Active Neuron Least Squares: A Training Method for Rectified Neural Networks
12:20–14:00	**Lunch**	
15:10–15:40	Chair: Woocheol Choi **Yoonmo Jung** AORC, Sungkyunkwan University, Korea	Optimization Models for Trend Filtering
15:50–16:20	Chair: Woocheol Choi **Donghwan Kim** KAIST, Korea	Two-time-scale gradient method for nonconvex-nonconcave minimax optimization
16:30–17:00	Chair: Woocheol Choi **Abdullah Alazemi** Kuwait University, Kuwait (Room: Diamond)	The Moore-Penrose inverse of symmetric matrices with nontrivial equitable partitions
16:30–17:00	Chair: Jang Soo Kim **Youngjoon Hong** AORC, Sungkyunkwan University, Korea (Room: Rose)	Invertible Monotone Operators for Normalizing Flows
17:10–17:40	Chair: Woocheol Choi **Ernest K. Ryu** Seoul National University, Korea	Continuous-Time Analysis of Accelerated Gradient Methods via Conservation Laws in Dilated Coordinate Systems
December 4, Sunday		
10:50–11:20	Chair: Abdullah Alazemi **Dragana Cvetković Ilić** Kuwait University, Kuwait	Solvability and different solutions of the operator equation $XAX = BX$

List of Abstracts – Talks

Plenary talks

Combinatorial Matrices and Their Continuous Analogues

Richard A. Brualdi

University of Wisconsin–Madison, USA

What is a **combinatorial matrix**? It's a generic term. There is no precise definition. Combinatorial matrices are simply matrices defined by a variety of combinatorial considerations or which have some combinatorial significance.

The simplest instances are $m \times n$ $(0,1)$-matrices, matrices each of whose entries is either a 0 or 1. Why are these considered to be combinatorial matrices? Such a matrix $A = [a_{ij}]$ can be regarded as a representation of a family $\mathcal{S}_m = (S_1, S_2, \ldots, S_m)$ of m subsets of a set X_n of n elements which can be assumed to be the set $X_n = \{1, 2, \ldots, n\}$ of the first n positive integers. Specific assumptions (combinatorial!) on these subsets are reflected in properties of the matrix A. The cardinality of the sets S_i equal the number of 1's in row i, the ith row sums r_i; the number of times an integer j occurs in the sets of the family $\mathcal{S}_m$ equals the number of sets containing the element j. Except for the labels given to the sets and their elements, the matrix A captures the family perfectly; each property of $\mathcal{S}$ can be stated in terms of A, and vice-versa.

Examples include: Permutation matrices, more generally, Multipermutation matrices; Adjacency Matrices of graphs, digraphs (e.g. tournaments), hypergraphs, and other matrices associated with graphs (e.g. Laplacians); Incidence matrices of combinatorial designs (e.g. block designs); Nonnegative integral matrices (e.g. incidence matrices of multisets); Matrices of 0's, 1's, and -1's, in particular, Alternating Sign Matrices (ASMs) and Adjacency Matrices of oriented graphs; etc.

Γ-graphic delta-matroids and their applications

Sang-il Oum

Discrete Mathematics Group, Institute for Basic Science, Korea
KAIST, Korea

Delta-matroids, introduced by Bouchet, are combinatorial objects capturing combinatorial properties of skew-symmetric or symmetric matrices. For an abelian group Γ, a Γ-labelled graph is a graph whose vertices are labelled by elements of Γ. We prove that a certain collection of edge sets of a Γ-labelled graph forms a delta-matroid, which we call a Γ-graphic delta-matroid, and provide a polynomial-time algorithm to solve the separation problem, which allows us to apply the symmetric greedy algorithm of Bouchet to find a maximum weight feasible set in such a delta-matroid. We present two algorithmic applications on graphs; Maximum Weight Packing of Trees of Order Not Divisible by k and Maximum Weight S-Tree Packing. We also discuss various properties of Γ-graphic delta-matroids. This is joint work with Duksang Lee.

Graph-theoretical Models of the Spread and Control of Disease and of Fighting Fires

Fred Roberts

Rutgers University, USA

We will describe irreversible threshold processes on graphs that model the spread of disease and lead to insights about strategies for vaccination, quarantine, etc.; and we will describe models of the control of fires that are mathematically analogous to the disease spread models. The analogy will lead to insights about both types of processes and a variety of challenging graph-theoretical problems.

Identifying Differential Equations with Numerical Methods: Time Evolution, Subspace Pursuit and Weak Form

Sung Ha Kang

Georgia Institute of Technology, USA

We consider identifying differential equation from one set of noisy observation. We assume that the governing equation can be expressed as a linear combination of different linear and nonlinear differential terms. This talk will cover collection of recent work on this topic: starting from numerical time evolution (IDENT), we consider robust identification methods using successively denoised differentiation and subspace pursuit, and consider recovery of kernel for nonlocal potential. We further discuss using weak form for ODE and PDE recovery. Using weak form shows robustness against higher level of noise and higher order derivative in underlying equation. This talk may include statistical analysis for this approach.

Smoothing strategies in optimization and data science

Jein-Shan Chen

National Taiwan Normal University, Taiwan

In this talk, we present smoothing strategies to some popular problems in optimization and data science, including support vector regression, signal reconstruction problem, and image deblurring. We demonstrate how smoothing functions can be involved or constructed to fit in these types of problems. In general, for each problem, it requires to employ some certain algorithm to work along with the proposed smoothing functions. Numerical experiments and comparisons affirm that our strategies are efficient approaches.

Kemeny's constant for an undirected graph: how much can adding one edge change things?

Steve Kirkland

University of Manitoba, Canada

Given a connected graph G, we consider the corresponding random walk, described as follows. Our random walker moves from one vertex to another in discrete time; when the walker is on vertex v at time t, one of the neighbours of v, say v', is chosen at random, and the walker moves to vertex v' at time t+1. The expected time it takes for the random walker to move from a randomly chosen initial vertex to a randomly chosen destination vertex is known as Kemeny's constant, and it can be thought of as measuring the random walker's ease of movement through the graph.

What happens to Kemeny's constant when a new edge is added to G? It turns out that depending on the structure of the graph and the placement of the new edge, Kemeny's constant might increase, or decrease, or stay the same. In this talk we take a step towards quantifying this behaviour. For each natural number n, we consider the family of trees on n vertices. We identify the tree in that family, as well as the new edge to be added, so that the increase in Kemeny's constant is maximized. We also solve the corresponding problem for maximizing the decrease in Kemeny's constant. The techniques rely on a detailed analysis of distance matrices for trees.

Tensor Decompositions

Grey Ballard

Wake Forest University, USA

Tensor decompositions are generalizations of low-rank matrix approximations to higher dimensional data. They have become popular for their utility across applications—including blind source separation, dimensionality reduction, compression, anomaly detection—where the original data is represented as a multidimensional array. We'll highlight a few applications where tensor decompositions, such as CP, Tucker, and Tensor Train decompositions, are particularly effective. We'll discuss properties of the various decompositions, and we'll describe the algorithms used to compute them.

Combinatorics and Matrix Theory

Characterization of equality in a perrank bound

Bryan Shader

University of Wyoming, USA

The *permanent rank* of a matrix A is denoted by perrank A and is defined to be the largest k such that A has a $k \times k$ submatrix with nonzero permanent. It is known that

$$\text{perrank } A \geq \text{rank } A/2.$$

We characterize the matrices for which equality holds.

Affine Gordon–Bender–Knuth identities for cylindric Schur functions

Jang Soo Kim

AORC, Sungkyunkwan University, Korea

The Gordon-Bender-Knuth identities are determinant formulas for the sum of Schur functions of partitions with bounded height, which have interesting combinatorial consequences such as connections between standard Young tableaux of bounded height, lattice walks in a Weyl chamber, and noncrossing matchings. In this talk we give an affine analog of the Gordon-Bender-Knuth identities, which are determinant formulas for the sum of cylindric Schur functions. We also consider combinatorial aspects of these identities. As a consequence we obtain an unexpected connection between cylindric standard Young tableaux and r-noncrossing and s-nonnesting matchings. This is joint work with JiSun Huh, Christian Krattenthaler, and Soichi Okada.

List 4-colouring of planar graphs

Xuding Zhu

Zhejiang Normal University, China

It is known that there are planar graphs G and 4-list assignments L of G such that G is not L-colourable. A natural direction of research is to put restrictions on the list assignments so that for any planar graph G and any 4-list assignment L of G satisfying the restrictions, G is L-colourable. One kind of lists studied in the literature is lists with separation. A (k,s)-*list assignment* of G is a k-list assignment of G with $|L(x) \cap L(y)| \leq s$ for each edge xy. A graph G is called (k,s)-choosable if G is L-colourable for any (k,s)-list assignment L of G. Mirzakhani constructed a planar graph G which is not $(4,3)$-choosable and Kratochvíl, Tuza and Voigt proved that every planar graph is $(4,1)$-choosable. A natural question (asked by Kratochvíl, Tuza and Voigt) is whether every planar graph is $(4,2)$-choosable. This question received a lot of attention, but there were not much progress. Recently, I proved that the answer to this quesiton is positive. In this lecture, I shall sketch the proof.

Mutually orthogonal binary frequency squares

Ian Wanless

Monash University, Australia

A *binary frequency square* is a square $(0, 1)$-matrix in which each row and column has the same number of zeroes in it. If each row and column has t ones and $n-t$ zeroes then we say the square has frequencies $(n-t, t)$. Two binary frequency squares F1 and F2 with frequencies (λ_0, λ_1) and (μ_0, μ_1) are *orthogonal* if for each $\ell, m \in \{0, 1\}$ there are $\lambda_\ell \mu_m$ ordered pairs $F_1[i, j], F_2[i, j] = (\ell, m)$.

In this talk I will survey some old and many new results on sets of mutually orthogonal binary frequency squares. Of particular interest will be sets that are *maximal* in that they cannot be extended to a larger set whilst preserving orthogonality. Several tools for diagnosing maximality have recently been developed.

On strong Sidon sets of integers

Sang June Lee

Kyung Hee University, Korea

Let $\mathbb{N}$ be the set of natural numbers. A set $A \subset \mathbb{N}$ is called a *Sidon set* if the sums $a_1 + a_2$, with $a_1, a_2 \in S$ and $a_1 \leq a_2$, are distinct, or equivalently, if

$$|(x + w) - (y + z)| \geq 1$$

for every $x, y, z, w \in S$ with $x < y \leq z < w$. We define strong Sidon sets as follows: For a constant α with $0 \leq \alpha < 1$, a set $S \subset \mathbb{N}$ is called an α-*strong Sidon set* if

$$|(x + w) - (y + z)| \geq w^\alpha$$

for every $x, y, z, w \in S$ with $x < y \leq z < w$.

The motivation of strong Sidon sets is that a strong Sidon set generates many Sidon sets by altering each element a bit. This infers that a dense strong Sidon set will guarantee a dense Sidon set contained in a sparse random subset of $\mathbb{N}$.

In this talk, we are interested in how dense a strong Sidon set can be. This is joint work with Yoshiharu Kohayakawa, Carlos Gustavo Moreira and Vojtěch Rödl.

Clones of pigmented words and realizations of varieties of monoids

Samuele Giraudo

Université Gustave Eiffel, France

Given a variety of algebras, that is a set of operations with their relations, a classical question is, given two compound operations, decide if they are equivalent. This is known as the word problem and in some cases, term rewrite systems provide solutions by orienting the relations in a suitable way. Here, we use combinatorial tools to address instances of this problem by encoding the equivalence classes of compound operations by combinatorial objects, compatible with the composition of operations. For this, we use clones, devices generalizing operads, used to represent varieties of algebras. We introduce a general combinatorial construction of clones involving pigmented words. As quotients, this clone contains several clones representing the varieties of some classes of monoids. Among others, we provide a combinatorial realization of regular band monoids.

The Permutation and Alternating Sign Matrix Cones

Geir Dahl

University of Oslo, Norway

This talk is on some work in combinatorial matrix theory (CMT), i.e., combinatorics related to matrix theory. We will look at two classes of matrices, the permutation matrices and the so-called alternating sign matrices (ASM). The goal of the talk is to give some basic facts on ASMs, mention a connection to physics, and to discuss how polyhedra (in particular, polyhedral cones) associated with permutation matrices and ASMs have some interesting properties. The talk will mostly be non-technical, and focus on concepts and main results. The talk is based on work done in collaboration with Richard Brualdi (Univ. Wisconsin, Madison).

An approximate version of the Strong Nine Dragon Tree Conjecture

Daqing Yang

Zhejiang Normal University, China

The fractional arboricity of a graph $G = (V, E)$, denoted by $\Gamma_f(G)$, is defined as $\Gamma_f(G) = \max_{H \subseteq G, |V(H)|>1} \frac{|E(H)|}{|V(H)|-1}$. The Strong Nine Dragon Tree Conjecture asserts that, for any non-negative integers k, d, if $\Gamma_f(G) \leq k + \frac{d}{k+d+1}$, then G decomposes into $k+1$ forests, with one of them consisting of subtrees of sizes at most d.

In this paper, we prove an approximate version of this conjecture. We prove that if $\Gamma_f(G) \leq k + \frac{d}{k+d+1}$, then G decomposes into $k+1$ forests, with one of them consisting of subtrees of sizes at most $\delta(d, k)$; where

$$\delta(d,k) = \begin{cases} d, & \text{if } d \leq k+1; \\ d + \frac{k}{k+1}d - k, & \text{if } k+1 < d \leq 2(k+1); \\ 2\frac{d}{k+1} + \frac{k}{k+1}\left\lceil \frac{d}{k+1} \right\rceil d, & \text{if } d > 2(k+1). \end{cases}$$

Note that in all the cases of $\delta(d, k)$, we have $\delta(d, k) \leq d^2$. This is a joint work with Yaqin Zhang.

Parallel Approximation Algorithm for the Minimum Submodular Cover Problem

Zhao Zhang

Zhejiang Normal University, China

In the minimum cost submodular cover problem (MinSMC), we are given a monotone nondecreasing submodular function $f : 2^V \mapsto \mathbb{Z}^+$, a linear cost function $c : V \mapsto \mathbb{R}^+$, and an integer $k \leq f(V)$, the goal is to find a subset $A \subseteq V$ with the minimum cost such that $f(A) \geq k$. The MinSMC can be found at the heart of many machine learning and data mining applications. In this paper, we design a parallel algorithm for the MinSMC that takes at most $O(\frac{\log km \log \gamma \log m}{\varepsilon^3})$ adaptive rounds, and it achieves an approximation ratio of $\frac{H(\min\{\gamma,k\})}{1-5\varepsilon}$ in expectation, where $\gamma = \max_{v \in V} f(v)$, $H(\cdot)$ is the Harmonic number, $m = |V|$, and ε is a constant in $(0, \frac{1}{5})$. All previous parallel algorithms for the MinSMC only work for the "unweighted" version, and the best know previous approximation ratio is $O(\log k)$. Our algorithm not only works for the weighted version, but also improves the approximation ratio from $O(\log k)$ to $O(\log \gamma)$. Note that in a worst case, $k = \Omega(n)$, and in general, γ is much smaller than n.

Matrix theory with Applications

A Roadmap for Solving Optimization on Stiefel Manifolds from Machine Learning

Ren-Cang Li

University of Texas at Arlington, USA
Hong Kong Baptist University, Hong Kong

Optimization problems on Stiefel Manifolds arise frequently in machine learning models. In this talk, we will present a roadmap via the NEPv (nonlinear eigenvalue problem with eigenvector dependency) approach to solve such problems.

Geometry of geometric mean and spectral mean

Tin-Yau Tam

University of Nevada, Reno, USA

Given two $n \times n$ positive definite matrices A and B, the metric geometric mean introduced by Pusz and Woronowicz in 1975 is

$$A\sharp B := A^{1/2}(A^{-1/2}BA^{-1/2})^{1/2}A^{1/2},$$

and the spectral geometric mean introduced by Fiedler and Pták in 1997 is

$$A\natural B := (A^{-1}\sharp B)^{1/2}A(A^{-1}\sharp B)^{1/2}.$$

For $0 \le t \le 1$, the t-*metric geometric mean* (t-geometric mean, for short) and t-*spectral geometric mean* (t-spectral mean, for short) of A and B are defined by

$$\begin{aligned} A\sharp_t B &:= A^{1/2}(A^{-1/2}BA^{-1/2})^t A^{1/2}, \\ A\natural_t B &:= (A^{-1}\sharp B)^t A(A^{-1}\sharp B)^t. \end{aligned}$$

Log majorization results involving t-metric geometric mean and t-spectral geometric mean are given and their geometry is explained. Some questions are asked.

Low Multi-Rank Third-order Tensor Completion by Riemannian Conjugate Gradient Descent Method

Guangjing Song

Weifang University, China

The goal of tensor completion is to fill in missing entries of a partially known tensor under a low-rank constraint. In this talk, we study low rank third-order tensor completion problems by using Riemannian optimization methods on the smooth manifold. Here the tensor rank is defined to be a multi-rank under the Fourier-related transform tensor-tensor product. With suitable incoherence conditions on the underlying low multi-rank rank tensor, we show that the proposed Riemannian optimization method is guaranteed to converge and it can find the underlying low multi-rank tensor with a high probability. Number of sampling entries required for convergence are also derived. Numerical examples for image data sets are reported to demonstrate that the performance of the proposed method is better than that of tensor based method using the Tucker-rank model in terms of computational time, and that of the matrix-based completion method in terms of number of sampling entries.

Solvability and the general solution to a Sylvester-type quaternion matrix equation with applications

Qing-Wen Wang

Shanghai University, China

In this talk, we mainly consider a Sylvester-type matrix equation

$$A_1X_1 + X_2B_1 + A_2Y_1B_2 + A_3Y_2B_3 + A_4Y_3B_4 = C_1$$

over the quaternion algebra. We give the solvability conditions and a formula of a general solution to the matrix equation. As applications, we also investigate the solvability conditions and the formulas of the general solutions to some other quaternion matrix equations.

Matrix Inequalities on Metric and Spectral Geometric Means

Luyining Gan

University of Nevada, Reno, USA

In this talk, we will introduce the relation between the weighted metric geometric mean and the weighted spectral geometric mean of the positive matrices in terms of log-majorization relation. In addition, we will also introduce some new properties of the weighted spectral geometric mean, like geodesic property and tolerance relation.

The complete equivalence canonical form of four matrices over the real quaternion algebra

Zhuo-Heng He

Shanghai University, China

In this talk, we give the complete structures of the equivalence canonical form of four matrices over the real quaternion algebra. As applications, we derive some practical necessary and sufficient conditions for the solvability to some systems of generalized Sylvester quaternion matrix equations using the ranks of their coefficient matrices. The results of this paper are new and available over the real number field and the complex number field.

On the rank problems of the third-order quaternion tensors

Yang Zhang

University of Manitoba, Canada

In this talk, we investigate the ranks of the third order quaternion tensors. We find the maximal ranks of $m \times 2 \times n$ quaternion tensors for any $m, n \in \mathbb{N}^+$ as well as $3 \times 3 \times 3$ quaternion tensors. We also explore some upper bounds of the ranks for higher-order quaternion tensors by using a block tensor approach. This is a joint work with Yungang Liang.

Optimization and Data Science

Two-time-scale gradient method for nonconvex-nonconcave minimax optimization

Donghwan Kim

KAIST, Korea

Nonconvex-nonconcave minimax problems in modern machine learning applications still remain difficult to optimize by a gradient-based method. One of the main reasons is its non-optimal limit points. This talk will first review the notion of local optimality in minimax optimization, and discuss the importance of time-scale separation in a gradient-based method for finding a local optimal point.

Active Neuron Least Squares: A Training Method for Rectified Neural Networks

Yeonjong Shin

KAIST, Korea

In this talk, we will present the Active Neuron Least Squares (ANLS), an efficient training algorithm for neural networks (NNs). ANLS is designed from the insight gained from the analysis of gradient descent training of NNs, particularly, the analysis of Plateau Phenomenon. The core mechanism is the option to perform the explicit adjustment of the activation pattern at each step, which is designed to enable a quick exit from a plateau. The performance of ANLS will be demonstrated and compared with existing popular methods in various learning tasks ranging from function approximation to solving PDEs.

Optimization Models for Trend Filtering

Yoonmo Jung

AORC, Sungkyunkwan University, Korea

Trend filtering is a regression problem to estimate underlying trends in time series data. It is necessary to investigate data in various disciplines. We propose a trend filtering method by adaptive piecewise polynomials. More specifically, we adjust the location and the number of breakpoints or knots to obtain a better fitting to given data. The numerical results on synthetic and real data sets show that it captures distinct features such as abrupt changes or kinks and provides a simplified form and brief summary of given data.

Remarks on the optimal control problems

Woocheol Choi

AORC, Sungkyunkwan University, Korea

In this talk, I will give a brief introduction for optimal control problems. I will explain related optimization problems and formulations of the problem for obstacle avoidance. A few technical issues for real time solvers of the optimal control problems will be introduced.

The Moore-Penrose inverse of symmetric matrices with nontrivial equitable partitions

Abdullah Alazemi

Kuwait University, Kuwait

In this paper we consider symmetric matrices that admit nontrivial equitable partitions. We determine some sufficient conditions for the quotient matrix of the Moore-Penrose inverse of the initial matrix to be equal to the Moore-Penrose inverse of its quotient matrix. We also study several particular cases when the computation of the Moore-Penrose inverse can be reduced significantly by establishing the formula for its computation based on the Moore-Penrose inverse of the quotient matrix. Among others we consider the adjacency matrix of a generalized weighted threshold graph.

Invertible Monotone Operators for Normalizing Flows

Youngjoon Hong

AORC, Sungkyunkwan University, Korea

Normalizing flows model probability distributions by learning invertible transformations that transfer a simple distribution into complex distributions. Since the architecture of ResNet-based normalizing flows is more flexible than that of coupling-based models, ResNet-based normalizing flows have been widely studied in recent years. Despite their architectural flexibility, it is well-known that the current ResNetbased models suffer from constrained Lipschitz constants. In this paper, we propose the monotone formulation to overcome the issue of the Lipschitz constants using monotone operators and provide an in-depth theoretical analysis. Furthermore, we construct an activation function called Concatenated Pila (CPila) to improve gradient flow. The resulting model, Monotone Flows, exhibits an excellent performance on multiple density estimation benchmarks (MNIST, CIFAR-10, ImageNet32, ImageNet64).

Continuous-Time Analysis of Accelerated Gradient Methods via Conservation Laws in Dilated Coordinate Systems

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We analyze continuous-time models of accelerated gradient methods through deriving conservation laws in dilated coordinate systems. Namely, instead of analyzing the dynamics of $X(t)$, we analyze the dynamics of $W(t) = t^{\alpha}(X(t) - X_c)$ for some α and X_c and derive a conserved quantity, analogous to physical energy, in this dilated coordinate system. Through this methodology, we recover many known continuous-time analyses in a streamlined manner and obtain novel continuous-time analyses for OGM-G, an acceleration mechanism for efficiently reducing gradient magnitude that is distinct from that of Nesterov. Finally, we show that a semi-second-order symplectic Euler discretization in the dilated coordinate system leads to an $\mathcal{O}(1/k^2)$ rate on the standard setup of smooth convex minimization, without any further assumptions such as infinite differentiability.

Solvability and different solutions of the operator equation $XAX = BX$

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We will present certain necessary and sufficient conditions for the existence of a non zero solution of the operator equation $XAX = BX$. The general form of the solution and describe all cases when this equation has infinitely many solutions as well as when all the solutions are idempotent will be given. We generalize the existing particular results that concern this equation in the special cases when $A = B$ or $A* \leq B$. Furthermore, we derive two possible representations of an arbitrary solution as a 2×2 operator matrix which allows us to discuss the existence of an invertible and a positive solution but also to open the discussion to the existence of any type of solutions using related results for 2×2 operator matrices. Also, we consider the operator equation $XAX = AX$ which is closely related to the "invariant subspace problem" and describe the set of all right(left) invertible and all Fredholm solutions.