Keynote Talk

Title Generalized numerical ranges and their applications

Speaker Chi-Kwong Li

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Abstract The numerical range is a usual tools for studying matrices and operators. There are many generalizations of the concept motivated by pure and applied topics. We will discuss some recent research results and new directions in this line of study.

Plenary Talks

Title Block Alternating Splitting Implicit Iteration Methods for Saddle Point Problems from Time-Harmonic Eddy Current Models

Speaker Zhong-Zhi Bai

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Abstract For the saddle-point problems arising from the finite element discretizations of the hybrid formulations of the time-harmonic eddy current problems, we establish a class of block alternating splitting implicit iteration methods and demonstrate its unconditional convergence. Experimental results are given to show the feasibility and effectiveness of this class of iterative methods when they are employed as preconditioners for Krylov subspace methods such as GMRES and BiCGSTAB.

Title Operator Inequalities of Löwner–Heinz Type

Speaker Mohammad Sal Moslehian

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Abstract Let $(\mathscr{H}, \langle\cdot,\cdot\rangle)$ be a complex Hilbert space and $\mathbb{B}(\mathscr{H})$ denote the algebra of all bounded linear operators on $\mathscr{H}$ equipped with the operator norm $\|\cdot\|$. An operator

$A \in \mathbb{B}(\mathscr{H})$ is called positive if $\langle Ax, x\rangle \geq 0$ holds for every $x \in \mathscr{H}$ and then we write $A \geq 0$. For self-adjoint operators $A, B \in \mathbb{B}(\mathscr{H})$, we say $A \leq B$ if $B - A \geq 0$. Further, we write $A > B$ if $A \geq B$ and $A - B$ is invertible. The relation $>$ is called strict positivity.

A continuous real valued function f defined on an interval J is called operator monotone if $A \geq B$ implies that $f(A) \geq f(B)$ for all self-adjoint operators A, B with spectra in J. The Löwner–Heinz inequality says that, $f(x) = x^r$ $(0 < r \leq 1)$ is operator monotone on $[0, \infty)$. It is equivalent to the Araki–Cordes inequality $\|AB\|^p \geq \|A^pB^p\|$ for all $A, B \geq 0$ and $0 \leq p \leq 1$.

In this talk, we investigate several norm inequalities corresponding to the Löwner–Heinz inequality, the Heinz inequality as well as some operator inequalities involving the strict positivity and operator monotone functions. We also find a lower bound for $f(A) - f(B)$, where f is a nonconstant operator monotone function. We also give an estimation of the Furuta inequality.

Title Spectral Inequalities and Quantum Marginal Problems

Speaker Yiu-Tung Poon

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Abstract The characterization of the spectra of Hermitian matrices A, B and C satisfying $A + B = C$ began with Weyl's inequalities in 1912. The problem was solved 15 years ago with conditions given by a set of linear inequalities. Recently, similar inequalities arise in the solution of some quantum marginal problems. However, in both cases, the number of inequalities grows exponentially with the size of the matrices. In this talk, we will discuss the connection among these and other related problems. In particular, we are interested in specific problems for which the solution can be given by a small set of inequalities.

Title Hankel Tensors and Circular Tensors

Speaker Liqun Qi

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Abstract Hankel tensors and circular tensors arise from applications such as signal processing. In this talk, we explore their special structures and proerties. These include

1. the associated Hankel matrix, the associated plane tensor, generating functions and Vandermonde decomposition of a Hankel tensor;
2. the root tensor, native eigenvalues and symmetrization of a circular tensor;
3. positive semi-definiteness and copositiveness of Hankel and circular tensors;
4. Hadamard products of Hankel and circular tensors;
5. specetral properties of Hankel and circular tensors.

Title A Backward Stable Algorithm for Quadratic Eigenvalue Problems

Speaker Yangfeng Su

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Co-authors Linghui Zeng

Abstract Quadratic eigenvalue problems (QEPs) appear in almost all vibration analysis of systems, such as buildings, circuits, acoustic structures, and so on. Conventional numerical method for QEPs is to linearize a QEP as a doublly-sized generalized eigenvalue problem (GEP), then call a backward stable algorithm to solve the GEP, for example, the QZ for dense GEP, and at last recover approximated eigenpairs of original QEP from those of the GEP.

However, the growth factor of the condition number in linearization, that is the ratio of the condition numbers between the QEP and the linearized GEP, may be much greater than 1, the growth factor of the backward error in recovery, that is the ratio of the backward errors between the recovered approximated eigenpairs of the QEP and the ones of the GEP, may also much greater than 1. To improve these growth factors, one needs to use a scaling before linearizations, carefully choose the linearizations, and properly recover the approximated eigenpairs. The FLV scaling by Fan, Lin and van Dooren can effectively improve the growth factors for not heavily damped QEP, the tropical scaling by Gaubert and Sharify can effectively improve the growth factors for heavily damped QEP with well-conditioned matrices.

In this talk, we give an algorithm for solving the complete solution of a QEP, and prove that the growth factors of condition numbers and backward errors are of order 1, and in turn, the algorithm is backward stable for all QEPs, no matter the QEP is heavily damped or not, no matter the matrices in QEP are well- or ill-conditioned.

Title Recent matrix asymptotic results and their Lie counterparts

Speaker Tin-Yau Tam

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Abstract: We will discuss some recent asymptotic results in matrix space and their extensions in Lie group, namely, (1) Beurling-Gelfand-Yamamoto's theorem and the generalization of Huang and Tam; (2) QR and Iwasawa asymptotic results of Huang and Tam, (3) Antezana-Pujals-Stojanoff convergence theorem on Aluthge iteration and the generalization of Tam and Thompson; (4) Rutishauer's LR algorithm and the generalization of Thompson and Tam; (5) Francis-Kublanovskaya's QR algorithm and the generalization of Holmes, Huang and Tam. These results are related to several important matrix decompositions, namely, SVD, QR, Gelfand-Naimark, Jordan and their counterparts namely, Cartan, Iwasawa, Bruhat, complete multiplicative Jordan decomposition.

Title A New Structure-Preserving Method for Quaternion Hermitian Eigenvalue Problems

Speaker Musheng Wei

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Co-authors Zhigang Jia, Sitao Ling

Abstract In this paper we propose a novel structure-preserving algorithm for solving the right eigenvalue problem of quaternion Hermitian matrices. The algorithm is based on the structure-preserving tridiagonalization of the real counterpart for quaternion Hermitian matrices by applying orthogonal JRS-symplectic matrices. The algorithm

is numerically stable because we use orthogonal transformations; the algorithm is very efficient, it costs about a quarter arithmetical operations, and a quarter to one-eighth CPU times, comparing with standard general-purposed algorithms. Numerical experiments are provided to demonstrate the efficiency of the structure-preserving algorithm.

Title Moving Convex Hulls for Sparse Recovery

Speaker Zhenyue Zhang

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Co-authors Jiayun Mao

Abstract A novel technique Moving Convex Hulls (MoCH) is proposed for sparse recovery in signal processing or compressive sensing with given noise level ε, based on a locally closed convex envelope of the signal cardinality function $\|x\|_0$ in an ℓ_∞-ball $\mathcal{B}(a, \alpha) = \{x : \ \|x - a\|_\infty \leq \alpha\}$. As a basic step of the MoCH, the new model itself can produce a good approximation to the exact signal, and MoCH provides an iterative rule to improve the approximation. As the existing ℓ_1-norm based sparse models, there is a similar error bound for the solution of the new model to the ideal sparsest signal $x_{0,\varepsilon}$. More importantly, there is an unbounded attracting domain $\mathcal{G}_0$ for a chosen in which the local convexifying can capture $x_{0,\varepsilon}$ as the solution. We characterize $\mathcal{G}_0$ as three parts: an unbounded convex polyhedron without conditions on the coefficient matrix A, a convex polyhedron when $x_{0,\varepsilon}$ is unique and a minimiser of residual over the ℓ_∞-ball, and a set under conditions on the approximate orthogonality of columns of A and sparsity of $x_{0,\varepsilon}$. Moreover, there is a sequence of disjointed subdomains $\{\mathcal{G}_k\}$ such that starting at a point in $\mathcal{G}_k$, the MoCH can theoretically recover the sparsest signal within at most k moving steps. Two iterative algorithms, successive fixed-point iterations (SFPI) and an ADMM algorithm, are given for solving the local convexifying problem, and updating rules of the ball are discussed. The MoCH is compared with other existing algorithms on several simulation examples and four real-world data sets for sparse recovery in the applications like image recovery, face recognition, image deblurring, and hyperspectral unmixing.

Invited Talks

Title Alternating Direction Method of Multipliers for Inverse Eigenvalue Problems

Speaker Zheng-Jian Bai

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Abstract In this talk, we propose the alternating direction method of multipliers for solving several kinds of inverse eigenvalue problems: the semidenite inverse quadratic eigenvalue problem with a partial eigen- structure, the nonnegative inverse eigenvalue problem with partial eigen-data, and the semidenite inverse eigenvalue problem with prescribed entries and partial eigendata. We also report some numerical tests to illustrate the eectiveness of the proposed method.

Title The inverse, rank and product of tensors

Speaker Changjiang Bu

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Abstract In this talk, we give some basic properties for the left (right) inverse, rank and product of tensors. The existence of order 2 left (right) inverses of tensors are characterized. We obtain some equalities and inequalities on the tensor rank. We also show that the rank of a uniform hypergraph is independent of the ordering of its vertices, and the Laplacian tensor and the signless Laplacian tensor have the same rank for odd-bipartite even uniform hypergraphs.

Title A Polynomial Eigenproblem Approach for General Joint Block Diagonalization

Speaker Yun-feng Cai

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Co-authors De-cai Shi, Shu-fang Xu

Abstract Joint Block Diagonalization (JBD) of a given Hermitian matrix set $\mathcal{A} = \{A_I\}_{i=0}^{p}$ is to find a nonsingular matrix W such that $W^H A_i W$ for $i = 0, 1, \cdots, p$ are all block diagonal matrices with the same prescribed block diagonal structure. General

JBD (GJBD) attempts to solve JBD without knowing the block diagonal structure. GJBD arises in Independent Subspace Analysis (ISA)/Blind Source Separation (BSS), and is more difficult than JBD. In this talk, we show that GJBD of $\{A_i\}_{i=0}^{p}\min_x$ is strongly connected with the eigeninformation of the associated matrix polynomial $P_{\mathcal{A}}(\lambda) = \sum_{i=0}^{p} \lambda^i A_i$. Under the assumption that $P_{\mathcal{A}}(\lambda)$ has only simple eigenvalues, a solvability theory for GJBD is established and the solutions of GJBD are characterized by the eigeninformation. Based on the established theory, a numerical method is proposed to solve GJBD. Numerical tests show that this method is not only feasible for exact GJBD, but also able to handle approximate GJBD to certain extend.

Title Zeros of two-sided quadratic quaternion polynomials

Speaker Lianggui Feng

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Co-authors Xuwei and Yaobo

Abstract The structure and computation are given for the zeros of a two-sided quadratic quaternion polynomial. In particular, the computation formulae and the sharp bound of essential number are established for the zeros of a two-sided quaternion polynomial of the form $u^2 + puq + r (p, q, r \in \mathbb{H})$. As application, some known results are improved and a conjecture presented by Janovská and Opfer is answered.

Title Variational deflation for nonlinear eigenvalue problem in resonant tunneling diodes

Speaker Weiguo Gao

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Co-authors Shengyao Xu

Abstract In this work we introduce a variational condition for different eigenpairs of a nonlinear eigenvalue problem arising from the simulation of resonant tunneling diodes. This leads to a variational deflation strategy which in principle can be extended to the nonlinear eigenvalue problems whose nonlinearity comes from the boundary conditions. Numerical experiments on both 1D and 2D examples show that Newton's method

associated with our proposed variational deflation sometimes achieves more eigenpairs which are missing from the original algorithms. This is joint work with Shengyao Xu.

Title Graph isomorphisms and Jordan isomorphisms for Hermitian matrices over simple Artinian rings

Speaker Li-Ping Huang

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Abstract Suppose that R = (R,-) is a simple Artinian ring with an involution - and $\mathrm{char}(R) \neq 2$, and $\mathcal{H}_n(R,-)$ is the set of $n \times n$ Hermitian matrices over R on the involution -. For $A, B \in \mathcal{H}_n(R,-)$, if $0 \neq A - M = A_1A_2$ where $A_1 \in r^{n\times 1}$ and $A_2 \in R^{1\times n}$, then A and B are said to be adjacent and denoted by $A \sim B$. For the vertex set $\mathcal{H}_n(R,-)$, by the adjacency relation $\sim$, $(\mathcal{H}_n(R,-),\sim)$, becomes a simple graph, which is called a Hermitian matrix graph over R. Applying the representation matrices over division rings and the geometry of Hermitian matrices over division rings, the graph isomorphisms for two Hermitian matrix graphs over simple Artinian rings are characterized. As applications, the algebraic form of Jordan isomorphisms for Hermitian matrices over simple Artinian rings is obtained.

Title Deblurring and sparse unmixing for hyperspectral images

Speaker Ting-Zhu Huang

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Abstract The main aim of this work is to study total variation regularization in deblurring and sparse unmixing of hyperspectral images. In the model, we also incorporate blurring operators for dealing with blurring effects, particularly blurring operators for hyperspectral imaging whose point spread functions are generally system dependent and formed from axial optical aberrations in the acquisition system. An alternating direction method is developed to solve the resulting optimization problem efficiently. According to the structure of the TV regularization and sparse unmixing in the model, the convergence of the alternating direction method can be guaranteed. Experimental results are reported to demonstrate the effectiveness of the TV and sparsity model and

the efficiency of the proposed numerical scheme, and the method is compared to the recent Sparse Unmixing via variable Splitting Augmented Lagrangian and TV method by Iordache *et al*.

Title Cluster algebras and some matrices with certain positivity

Speaker Fang Li

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Co-authors Yichao Yang

Abstract In this talk, we discuss some matrices with certain positivity, which are: totally positive matrices, P-matrices, positive definite matrices. The method is introduced for these matrices to be parametrized and tested through planar networks. Moreover, the interpretation of these matrices is given via either cluster algebras or cluster subalgebras. This talk is a joint work with Yichao Yang.

Title Efficient Arnoldi-type Methods for Quadratic and Rational Eigenvalue Problems

Speaker Tiexiang Li

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Co-authors W.-Q. Huang, Y.-T. Li and W.-W. Lin.

Abstract In this paper, we are concerned with the computation of a few eigenpairs with smallest eigenvalues in absolute value of quadratic eigenvalue problems. We first develop a semiorthogonal generalized Arnoldi method where the name comes from the application of a pseudo inner product in the construction of a generalized Arnoldi reduction for a generalized eigenvalue problem. The method applies the Rayleigh-Ritz orthogonal projection technique on the quadratic eigenvalue problem. Consequently it preserves the spectral properties of the original quadratic eigenvalue problem. Furthermore, we propose a refinement scheme to improve the accuracy of the Ritz vectors for the quadratic eigenvalue problem. Given shifts, we also show how to restart the method by implicitly updating the starting vector and constructing better projection subspace. We combine the ideas of the refinement and the restart by selecting shifts upon the information of refined Ritz vectors. Finally an implicitly restarted refined

semiorthogonal generalized Arnoldi method is developed. Numerical examples demonstrate that the implicitly restarted semiorthogonal generalized Arnoldi method with or without refinement has superior convergence behaviors than the implicitly restarted Anoldi method applied to the linearized quadratic eigenvalue problem.

Title The Theory and Numerical Analysis for the Limiting Probability Vector of a Transition Probability Tensor

Speaker Wen Li

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Co-authors Lu-Bin Cui, Michael Ng

Abstract In this talk, we establish theoretical analysis for a limiting probability distribution vector of a transition probability tensor P arising from a higher order Markov chain, and then propose and develop an iterative method to calculate this vector. In the model, the computation of such limiting probability distribution vector x can be formulated as a $\mathbf{Z}$-eigenvalue problem $P_x^{m-1} = x$ associated with the eigenvalue 1 of P where all the entries of x are required to be non-negative and its summation must be equal to one. We show that if P is a transition probability tensor, then solutions of this $\mathbf{Z}$–eigenvalue problem exist. With some suitable conditions of P, the limiting probability distribution vector is unique. Under the same uniqueness assumption, the linear convergence of the iterative method and perturbation analysis of the limiting probability vector can be established. Numerical examples are presented to illustrate the theoretical results.

Title The Schur Complements of Block Diagonally Dominant Matrices and Their Disc Separation

Speaker Jianzhou Liu

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Abstract In this talk, we firstly consider the block dominant degree for I- (II-) block strictly diagonally dominant matrices and their Schur complements, showing that the block dominant degree for the Schur complement of an I-(II-) block strictly diagonally dominant matrix is greater than that of the original grand block matrix. Then, as application, we present some disc theorems and some bounds for the eigenvalues of the Schur complement by the elements of the original matrix.

Title Some Results of Condition Number of Drazin inverse

Speaker Xiaoji Liu

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Abstract Some results on the condition number for the Drazin inverse of a matrix A with regard to any unitary norm are derived. Various normwise relative condition numbers which measure the sensitivity of the solution of singular linear systems are also considered. We also consider the minimum quality of condition number of Drazin inverse. Finally, upper bounds for the sensitivity of structured perturbation are derived. A numerical example is presented for illustrating the result.

Title Quantum Error Correction with Mixed State Ancilla Qubits

Speaker Mikio Nakahara

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Abstract We provide the simplest yet interesting examples of operator quantum error corrections avoiding a class of fully correlated noise operators. Our schemes require no initialization of ancilla qubits, and hence they can be in any mixed state. Quantum correlations involved are also analyzed. We demonstrate one of our schemes experimentally by making use of a three-qubit NMR quantum computer. Possible extensions will be also discussed.

This talk is based on C.-K. Li, M. Nakahara, Y.-T. Poon, N.-S. Sze and H. Tomita, Physics Letters A P375 (2011) 3255-3258 and Y. Kondo, C. Bagnasco and M. Nakahara, Physical Review A 88 (2013) 022314 1-5.

Title Spectral data and solvent theory for regular matrix polynomials

Speaker Edgar Pereira

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Co-authors Nir Cohen, nir@ccet.ufrn.br, Universidade Federal de Rio Grande do Norte, Brazil

Abstract We present an extension of the fundamental theorem of algebra (roots and factorization) from complex polynomials to complex matrix polynomials, i.e. expressions of the form

$$P(\lambda) = \sum_{i=0}^{k} A_i \lambda^i \qquad A_i \in M_{n,m}(\mathbb{C}), \quad \lambda \in \mathbb{C}.$$

Presenting a re-evaluation of the spectral approach and factorizability for regular matrix polynomials.

Our objective is to integrate the solvent approach in a non-trivial way to a unique theory which is able to describe all the *regular* first order right factors of a matrix polynomial.

We use the concept of *bi-solvent*, related with the algebraic relation

$$\sum A_i S_1^i S_2^{k-i}.$$

The classification of bi-solvents is shown to be equivalent to the classification of all the regular first order right factors for a regular matrix polynomial, and it is done in terms of certain restrictions of standard pairs.

Some numerical examples illustrate the presented theory.

Title Determinantal and eigenvalue inequalities for matrices with numerical ranges in a sector

Speaker Nung-Sing Sze

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Abstract Let $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \in M_n$, where $A_{11} \in M_m$ with $m \leq n/2$, be such that the numerical range of A lies in the set $\{e^{i\varphi} z \in \mathbb{C} : |\Im z| \leq (\Re z) \tan \alpha\}$, for

some $\varphi \in [0, 2\pi)$ and $\alpha \in [0, \pi/2)$. We obtain the optimal containment region for the generalized eigenvalue λ satisfying

$$\lambda \begin{bmatrix} A_{11} & 0 \\ 0 & A_{22} \end{bmatrix} x = \begin{bmatrix} 0 & A_{12} \\ A_{21} & 0 \end{bmatrix} x \quad \text{for some nonzero } x \in \mathbb{C}^n,$$

and the optimal eigenvalue containment region of the matrix $I_m - A_{11}^{-1}A_{12}A_{22}^{-1}A_{21}$ in case A_{11} and A_{22} are invertible. From this result, one can show $|\det(A)| \leq \sec^{2m}(\alpha)|\det(A_{11})\det(A_{22})|$. In particular, if A is a accretive-dissipative matrix, then $|\det(A)| \leq 2^m|\det(A_{11})\det(A_{22})|$. These affirm some conjectures of Drury and Lin.

This talk is based on a joint work with and C.K. Li (College of William & Mary).

Title Parallel Multisplitting Two-stage Iterative Methods with General Weighting Matrices for Non-Hermitian Positive Definite Systems

Speaker Chuan-Long Wang

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Abstract In this work, we propose a new parallel multisplitting iterative method for non-Hermitian positive definite linear systems. Based on optimization theory, the new method has two great improvements: one is that only one splitting needs to be convergent, and the other is that the weighting matrices are not scale and nonnegative. Convergence is discussed. Finally, the numerical results show that the method is feasible.

Title ADDA and Deflation Technique for M-matrix Algebraic Riccati Equations

Speaker Wei-guo Wang

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Co-authors Wei-chao Wang, Ren-Cang Li,University of Texas at Arlington

Abstract A new doubling algorithm—the alternating-directional doubling algorithm (ADDA)—is developed for computing the unique minimal nonnegative solution of an

M-matrix algebraic Riccati equation (MARE). It is argued by both theoretical analysis and numerical experiments that ADDA is always faster than two existing doubling algorithms: SDA of Guo, Lin, and Xu (Numer. Math., 103 (2006), pp. 393-412) and SDA-ss of Bini, Meini, and Poloni (Numer. Math., 116 (2010), pp. 553-578) for the same purpose. Also demonstrated is that all three methods are capable of delivering minimal nonnegative solutions with entrywise relative accuracies as warranted by the defining coefficient matrices of a MARE. It is explained that ADDA is the best among all possible doubling algorithms resulted from all bilinear transformations. A deflation technique is then presented for an irreducible singular MARE. The technique improves the rate of convergence of a doubling algorithm, especially for an MARE in the critical case for which without deflation the doubling algorithm converges linearly and with deflation it converges quadratically. The deflation also improves the conditioning of the MARE in the critical case and thus enables its minimal nonnegative solution to be computed more accurately.

Title Parallel Multisplitting Iteration Methods Based on Optimization for Linear Systems

Speaker Rui-Ping Wen

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Abstract As we know, in a standard multisplitting the k-th iteration $x^{(k)}$ is chosen in the set $\{x|x=\sum_{i=1}^{m} E_i x_i^{(k)} \text{and} \sum_{i=1}^{m} E_i = I\}$ and all the splittings must be convergent. In this report, we want to find the nearest point to the solution of $Ax = b$ in the hyperplane $\{x_i^{(k)}, i = 1, 2, \cdots, m\}$ mg (even larger set). Hence, the optimization models are studied. Here, two models are presented in the following.

(a) When A is a symmetric positive definite matrix,

$$\min_x \frac{1}{2} x^T Ax - x^T b$$

$$s.t.\ x = \sum_{i=1}^{m} E_i x_i^{(k)}.$$

(b) When A is a non-symmetric positive definite matrix,

$$\min_{x}(Ax-b)^T MM^{-1}(Ax-b)x - x^T b$$

$$s.t.\ x = \sum_{i=1}^{m} E_i x_i^{(k)}.$$

Based on the optimization models (a) and (b), we prove that the multisplitting parallel iteration methods is convergent under weak conditions: (1) the weighting matrices $E_i,\ i=1,2,\cdots,m$ are not necessarily nonnegative and be given in advance; (2) only one of all the splittings $A = M_i - N_i, i = 1,2,\cdots,m$ is required to be convergent.

In view of computation, though the great time is taken in the solving the (a) (or (b)) at each step, the total iteration numbers will be decreased largely. On the other hand, the weighting matrices $E_i, i = 1,2,\cdots,m$ are chosen few nonzero variables and many zero entries, so that (a) (or (b)) is solved conveniently. Some numerical experiments show that these methods are effective.

Title Completely Positive Matrices whose cpranks equal ranks

Speaker Changqing Xu

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Co-authors Prof. Wasin So of San Jose State University, California.

Abstract This is a joint work with Prof. Wasin So of San Jose State University, California. An $n \times n$ nonnegative matrix A is called completely positive, if it can be decomposed as $A = BB^T$ where B is a entrywise nonnegative matrix. This is also equivalent to say that A is a Gramian matrix of n nonnegative vectors of some dimension m. As the dual cone of the copositive matrices which are defined as the symmetric matrices $M \in R^{n\times n}$ satisfying $x^T Mx \geq 0$ for all the nonnegative vectors $x \in R^n_+$, , Completely Positive matrices, as a kind of positive matrices, have been one of the focus in matrix theory since 1960s when Diananda, M.Hall Jr. etc.

began to use it to study quadratic forms and block designs in combinatorics. The most in uential period of cp problem shall be around 1990s when Avi Berman and his group (including Danny Hershkowitz, Noami Shaked-Monde, etc.) began to investigate cp by graph theory and its combinatorial meanings. The application of cp problem in data clustering and image analysis was found by A. Shashua etc. in 2005 and Chris Ding etc. in 2006. Recently, it is also found related to mathematical programming by K. Anstreicher, Bomze, Duer etc. Our interest in these work focus on the subset of a special cp matrices, i.e., cp matrices whose cpranks are exactly its ranks (Generally a cp matrix has a cprank larger than its rank). We present several easy-to-check sufficient conditions for a doubly nonnegative matrix to be cp with $cprank(A) = rank(A)$.

Title Some remarks on the representation of the Drazin inverse

Speaker Qingxiang Xu

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Abstract The Drazin inverse is a kind of generalized inverse, which has various applications. In this talk, we will focus on the representation of the Drazin inverse and its application in the study of the second-order homogeneous algebraic differential equations.

Title Complex nonsymmetric algebraic Riccati equation and its application in stochastic fluid flow models

Speaker JunGong Xue

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Abstract We present a class of complex nonsymmetric algebraic Riccatti equations (CNARE) and design numerical methods for its solutions. Based on the solutions of the CNAREs we obtaind a decomposition of the key matrices in the transient analysis for Markov-modulated fluid flow modes. With this kind of decomposition we present a new approach to carry out the transient analysis.

Title The general solution for a symmetric stochastic inverse eigenvalue problem

Speaker Shang-Jun Yang

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Abstract For a square matrix A, let $\sigma(A)$ denote the spectrum of A . Given an n-tuple $\Lambda = (\lambda_1, \cdots, \lambda_n)$ of numbers, the problem of deciding the existence of an $n \times n$ matrix A (of certain class) with $\sigma(A) = \Lambda$ is called the inverse eigenvalue problem (of certain class) which has long time been one of the problems of main interest in the theory of matrices. For a real n-tuple Λ, if there is a symmetric stochastic matrix whose spectrum is Λ, then we say that the symmetric stochastic inverse eigenvalue problem (mentioned as SSIEP) with Λ has a solution A.

Usually the theory of SSIEP studies under what condition satisfied by a given real n-tuple Λ the SSIEP with Λ has a solution (only one solution is needed). Here we present the concept of general solution (meaning all solutions, see Definition 1) of a SSIEP (of any order) and the concept of totally general solution of a 3×3 SSIEP (see Definition 2). Given a real triple Λ we firstly give the sufficient and necessary conditions for the 3×3 SSIEP with to have the totally general solution and give the formula of the totally general solution. Then we give the sufficient and necessary conditions for the 3×3 SSIEP to have the general solution and give the formula of the totally general solution. Finally we give the sufficient condition for a 4×4 SSIEP to have a solution set $A(x)$ with a parameter x, $x \in [0, 1]$.

The following are the main results:

Definition 1. The general solution of the SSIEP for $\Lambda = (\lambda_1, \cdots, \lambda_n), n \geq 2$ is a set of $n \times n$ symmetric stochastic matrices such that each element of ω is a solution of the SSIEP and each solution of the SSIEP is permutationally similar to some element of Ω.

Theorem 1. *Any solution of the* 3×3 *SSIEP for any* $\Lambda = (\lambda_1, \lambda_2, \lambda_3)$ *must be per-*

mutationally similar to $A(x) = S_3(x)diag(\lambda_1, \lambda_2, \lambda_3)S_3^T(x)$ *for some* $x \in [0, 1]$, *where*

$$S_3(x) = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2(x^2+x+1)}} & \frac{2x+1}{\sqrt{2x+16(x^2+x+1)}} \\ \frac{1}{\sqrt{3}} & -\frac{x+1}{\sqrt{2(x^2+x+1)}} & \frac{1-x}{\sqrt{2x+16(x^2+x+1)}} \\ \frac{1}{\sqrt{3}} & \frac{x}{\sqrt{2(x^2+x+1)}} & -\frac{x+2}{\sqrt{2x+16(x^2+x+1)}} \end{pmatrix} \tag{1}$$

Definition 2. If $A(x) = S_3(x)diag(\lambda_1, \lambda_2, \lambda_3)S_3^T(x)$ for each $x \in [0, 1]$ is a solution of the SSIEP for $\Lambda = (\lambda_1, \cdots, \lambda_n)$, then the bunch of matrices $A(x), x \in [0, 1]$ is called the totally general solution of the SSIEP.

Note that the general solution Ω of the SSIEP for a given real triple $\Lambda = (\lambda_1, \cdots, \lambda_n)$ is a subset of $A(x) = S_3(x)diag(\lambda_1, \lambda_2, \lambda_3)S_3^T(x)$, $x \in [0, 1]$ and they may not be equal in other words, the totally general solution of the SSIEP is the largest possible general solution of the SSIEP, and the general solution of the SSIEP for Λ is totally general solution of the SSIEP if and only if its domain is $[0, 1]$.

Theorem 2. The totally general solution of the SSIEP for $\Lambda = (\lambda_1, \lambda_2, \lambda_3)$, $1 \geq \lambda_1 \geq \lambda_2 \geq \lambda_3 \geq -1$ exits if and only if $1 + 2\lambda_3 \geq 0$, $2 - 3\lambda_2 + \lambda_3 \geq 0$ and the totally general solution is

$$A(x) = \frac{1}{3\alpha} \begin{pmatrix} \alpha + 3\lambda_2 + (1 + 2x)^2\lambda_3 & \alpha - 3(x + x)\lambda_2 + (1 + x - 2x^2)\lambda_3 & X \\ * & \alpha + 3(1 + x)^2\lambda_2 + (1 - x)^2\lambda_3 & Y \\ * & * & Z \end{pmatrix} \tag{2}$$

with $X = \alpha + 3x\lambda_2 - (2 + 5x + 2x^2)\lambda_3$, $Y = \alpha - 3(x + x^2)\lambda_2 - (2 - x - x^2)\lambda_3$, $Z = \alpha + 3x^2\lambda_2 + (2 + x)^2\lambda_3$, $\alpha = 2x^2 + 2x + 2$, $x \in [0, 1]$.

Theorem 3. The general solution of the SSIEP for $\Lambda = (1, \lambda_2, \lambda_3)$, $1 \geq \lambda_2\lambda_3 \geq -1$ exits if and only if $2 + \lambda_2 + 3\lambda_3 \geq 0$, the general solution is also given by (2), but the domain D of $A(X)$ is determined as follows:

(a) $D = [0, 1]$ when $\min\{1 + 2\lambda_3,\ 2 - 3\lambda_2 + \lambda_3\} \geq 0$;

(b) $D = [v, 1]$ when $1 + 2\lambda_3 \geq 0$ and $2 - 3\lambda_2 + \lambda_3 < 0$, where

$$v = \frac{-2 + 3\lambda_2 - \lambda_3 + \sqrt{-3(2 - 3\lambda_2 + \lambda_3)(2 + \lambda_2 - 3\lambda_3)}}{4(1 - \lambda_3)} \in (0, 1];$$

(c) $D = [u, 1]$ when $1 + 2\lambda_3 < 0$ and $2 - 3\lambda_2 + \lambda_3 < 0$, where

$$v = \frac{-1 - 2\lambda_3 + \sqrt{-3(1 + 2\lambda_2)(1 + 2\lambda_3)}}{2 + 3\lambda_2 + \lambda_3} \in (0, 1].$$

Title Inverse invariant zero-nonzero patterns

Speaker XingZhi Zhan

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Abstract We characterize the irreducible inverse invariant zero-nonzero patterns. To do so, we also characterize the transposed inverse invariant zero-nonzero patterns.

Title The algebraic connectivity of graphs

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Abstract Let G be a simple graph of order n and $L(G) = D(G) - A(G)$ be its Laplacian matrix, where $D(G)$ and $A(G)$ are the degree diagonal and adjacency matrices, respectively. The the second smallest eigenvalue of $L(G)$ is called the algebraic connectivity of G. In this talk, we survey some new results and progress on the algebraic connectivity. In particular, we present some relationships between the algebraic connectivity and the graph parameters, such as the clique number, the matching number, the independence number, the isoperimetric number, etc.

Title Computing Popov Forms of Matrices over Ore Domains

Speaker Yang Zhang

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Abstract In this talk we will discuss (weak) Popov forms of matrices over Ore domains, and exhibit effective algorithms to find them. In particular, we consider matrices over skew polynomial rings, Weyl algebras and Poincaré-Birkhoff-Witt extensions. As applications we give general methods to calculate the ranks of such matrices, and a method to transfer a system of differential equations into a first order equation.

Title The Semi-convergence of Uzawa-type iterative methods for Solving Singular Saddle Point Problems

Speaker Bing Zheng

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Abstract In this talk, we summerize the recent results about the semi-convergence of Uzawa-type iterative methods for solving singular saddle point problems. These results mainly come from [Bing Zheng, Zhong-Zhi Bai and Xi Yang, LAA, 2009], [Naimin Zhang, Tzon-Tzer Lu and Yimin Wei, JCAM, 2014]] and et.al..

Contributed talks

Title A pair of generalized Sylvester matrix equations

Speaker Zhuo-Heng He

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Abstract In this talk, we derive necessary and sufficient solvability conditions for the system of mixed generalized Sylvester matrix equations $A_1X-YB_1=C_1, A_2Y-ZB_2=C_2$, and give an expression of the general solution to this system when it is solvable. We also investigate the admissible ranks of the solution, and the admissible ranks and inertias of Hermitian part of the solution, respectively. As an application of the above system, we obtain solvability conditions and the general Hermitian solution to the generalized Sylvester matrix equation. Moreover, we provide an algorithm and a numerical example to illustrate our results.

Title A Relaxed Dimensional Factorization preconditioner for the incompressible Navier-Stokes equations(example)

Speaker Qiang Niu

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Abstract we discuss a class of Relaxed Dimensional Factorizations preconditioner for saddle point problems. Properties of the preconditioned matrix are analyzed and

compared with those of the closely related Dimensional Splitting (DS) preconditioner recently introduced by Benzi and Guo . Numerical results for a variety of finite element discretizations of both steady and unsteady incompressible how problems indicate very good behavior of the RDF preconditioner with respect to both mesh size and viscosity.

Title Research on some classical systems of matrix equations with applications

Speaker Guangjing Song

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Abstract Special solutions to system of matrix equations are not only key part of the matrix equation theory, but also play an important role in the studying of comparison of linear normal experiment. By the existing methods, we can derive the Hermitian solutions, the nonnegative definite solutions and the least norm solutions to system of matrix equations $AX = B,\ XC = D$. However, we can not get the real nonnegative definite solutions to this system.

In this report, we will add some different constrict conditions to the variant matrix in the expression of the general solution, then the problem of searching the real non-negative definite solutions to this system can be changed to finding some nonnegative definite solutions to a new system. Then the open problem can be solved. Noting that the Unitary solutions and some special solutions to system play important roles in linear model, we will show their applications in the comparison of linear experiment.

Title Learning algorithm with momentum and its applications

Speaker Naimin Zhang

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Abstract Two steepest descent algorithms with momentum for quadratic functions are considered. For a given learning rate, the sufficient and necessary conditions for the semi-stability of the steepest descent algorithms with momentum are established, respectively. Moreover, the optimal momentum factors which generally make for the fastest convergence are obtained.